The Ant colony Pseudocode for Mean-Variance-CVaR model of Multi-Portfolio Optimization

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Abstract:

The portfolio selection problem refers to form a good portfolio. It is complicated to choose which assets should be selected because of the doubt on their returns. In this paper we present a adaptive Mean-Variance-CVaR (MVC) model of multi-portfolio optimization . The present study uses to optimize the portfolio problem via ACO approach. To this idea, after we explain mathematical formulation of the problem, we will present a mew Simulated Pseudocode. It helps to recognize the details of the problem and finally Ant Colony procedure will propose to solve the MVC problem. ACO optimization leads to making the simplified and dependable and solvable problem.

Keywords: Ant colony, Multi-objective, portfolio optimization, Pseudocode

1. Introduction

The portfolio selection problem refers to form a good portfolio. It is complicated to choose which assets should be selected because of the doubt on their returns. The central purpose in a portfolio selection Problem is to find optimal proportions of the stock for creating a portfolio which complements the investor's preferences presumptuous that the investors' wish to strike a balance between maximizing the return and minimizing the risk of their investment [1].
The aim of solving the multi-objective portfolio optimization problem is finding the set of Pareto optimal that in this set we can find the balancing point of minimum risk and maximum return [7]. We can see the details of an objective hierarchy [3] from the classical Markowitz model for multi criteria decision making.

The rest of this paper is as the following. In next section explain the multi-objective portfolio optimization based on mathematical approach. Then discuss about multi-objective portfolio optimization based on ACO approach based on literature and recent studies. Next we present the MVC model of portfolio optimization based on ACO. Finally, results and some future works are presented at the conclusion.

2. Multi-Objective Portfolio Optimization

Let $R_1, R_2, \ldots, R_n$ be stochastic return rates of assets $1,2,\ldots,n$. We suppose that $E[|R_i| < \infty]$ for all $i = 1,2,\ldots,n$. Our plan is to invest our capital in these assets in order to get some attractive characteristics of the total return rate on the investment. Denoting by $x_1, x_2, \ldots, x_n$ the fractions of the original capital invested in assets $1,2,\ldots,n$ we can quickly gain the formula for the whole return rate:

$$R_1 x_1 + R_2 x_2 + \cdots + R_n x_n \qquad (1)$$

Obviously, the set of feasible asset allocations can be determined as follows [2], [3]:

$$X = \{x \in \mathbb{R}^n : x_1 + x_2 + \cdots + x_n = 1, x_i \geq 0,\ i = 1,2,\ldots,n\} \qquad (2)$$

Two criteria are important for portfolio optimization. They are expected returns and risks while the aim of investor is maximize the first one and minimize the second one. However, increase the returns usually led to increased risk. The main method that tries to solve the portfolio optimization problem introduced by Markowitz as mean –variance portfolio model [4]. The “efficient frontier” that obtained from this model show illustrates the level of minimum risk with maximum expected return as the optimal strategy [5], [6].

In the real time the problem of portfolio optimization is included the several items as constraints such as limitations of trading, portfolio size, etc. However, the primary Markowitz mean-variance model does not include them [5]. Nowadays, helping the power of decision in variable cases causes to the optimization techniques have a large using in financial field [7].

3. Multi-portfolio optimization via ACO approach

In ACO generally we see that algorithm starts by initialization of pheromone values. The next step is solution of problem of probabilistic method. After that pheromone must be updated in this step. The more details will bring in the next section as step by step to make of ACO algorithms for financial using.

To sum up, according to literature ACO procedure has four steps that they are listed:

Step1. Initialized by Ant

In this step generates the Ant colony. Equation of (3) use for select the fragment i from between the K possible choices. Note that τ_k is the amount of pheromone related with fragment k [12].

$$prob_i = \frac{\tau_i}{\sum_1^k \tau_k} \qquad (3)$$

Step2. Evaluate

This step includes an evaporation phase (let the evaporation rate is denoted by γ), and a pheromone deposit phase:

$$\tau_i(t+1) = \tau_i(t)(1-\gamma) + \delta_i \qquad (4)$$

During the update process the quantities of pheromone deposited on each solution has direct relation to the operation of the algorithm [12].

Step3. Ant distribution

In this step ants distribute according to their distance.

Step4. Stopping the iteration

Repeat the last process till reach the maximum numbers of ants or lack of optimal solutions.

4. MVC Model based on ACO approach

We face many decisions making in the real world. There are numerous methods for optimization of them. The aim of solving is finding the Pareto optimal solutions. However, in the theoretical problem cases, if Pareto set normally cannot be solved by an algorithm, then we try to approximate of Pareto sets [8].

One of the main methods of solving the multi-objective problems is using via algorithms (e.g. Genetic, Bee, Swarm and ACO algorithm). The approach of this study is ACO. This technique introduced by MarcoDorigo (1992) in his PhD thesis. The first ACO algorithm used for finding the best path in a graph [9].

Other researchers developed his method to find the optimal solution for other problems. For example, Karl et al. introduced one meta-heuristic method to find the Pareto optimal for portfolio optimization based on an ACO algorithm (for more examples see [10]) [11].

In this section we present an MVC model based on ACO with three approaches as the subsections. First we explain the problem via **mathematical formulation**. Next, we turn that formula to **Simulated Pseudocode**. Finally**,** we present one Ant Colony **Algorithm** to solve this problem.

4.1. Mathematical formulation

MVC uses the three parameters (: the expected value (E), the variance (σ^2) and the CVaR (conditional value at risk) at a specified confidence level α∈(0,1)) to better modeling. The aim of the proposed model is given a developed method in the solution [14].

We can obtain a better model of portfolio by replacing the three indexes instead of two parameters as the usual and it's led to increase the power of the model [15].

Several studies on portfolio optimizations via mathematical approaches have been investigated in the literature. In this paper, a MVC Model for the portfolio is given via ACO approach. To start of this model, we consider some variable, term and objective functions as the following nomenclature [10].

Let

N the number of assets that are available

R_x = be a return (as a random variable) depending on a decision vector x that belongs to a feasible set A

x_i = amount of investing on i th asset

Ω = the feasible set of solutions, or search space

s = a solution of problem

μ_i the expected mean of the i th asset

$\sigma^2(R_x)$ = the variance belong to R_x

K the number of assets to invest ($K \leq N$)

We define the variable

$$z_i = \begin{cases} 1 & \text{if the } i\text{th } (i = 1, \ldots, N) \text{ asset is chosen} \\ 0 & otherwise \end{cases} \quad (5)$$

x_i = money ratio ($0 \leq x_i \leq 1$) invested in the ith ($i = 1, \ldots, N$) asset

We consider the MVC model based on Aboulaich et al. (2010) method with following formula [14]:

$$\begin{cases} \min\{CVaR, -E, var\}, \\ s.t \quad x \in \left\{(x_1, \ldots, x_n) \middle| \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \forall j \in \{1, \ldots, n\}\right\}. \end{cases} \quad (6)$$

We note that, for random variable the value of variance is calculated by below formula:

$$\sigma^2(R_x) = E[(R_x - E(R_x))^2]. \quad (7)$$

Note that CVaR is calculated according to following theorem*:*

Proposition 3.1 (CVaR calculation and optimization): Let R_x be a random variable depending on a decision vector x that belongs to a feasible set A, and $\alpha \in (0,1)$. Consider the function:

$$F_\alpha(x, v) = \frac{1}{\alpha} E\{[-R_x, +v]^+\} - v, \quad (8)$$

where $[u]^+ = u$ for $u \geq 0$ and $[u]^+ = 0$ for $u < 0$. Then a function of v, F_α is finite and continuous (hence convex) and

$$CVaR(F_x) = \min_{v \in R} F_\alpha(x, v) \quad (9)$$

Thus, if the set A of feasible decision vectors is convex (which is the case for the basic version of the portfolio selection problem), and even if we impose a further lower limit on the expected return, minimizing CVaR is a convex optimization problem [15]. ■

$$\min\left\{\alpha + \frac{1}{m(1-\beta)} \sum_{k=1}^{m} u_k, -w^T\mu, w^T\sigma w\right\}, \quad (11)$$

and

$$\begin{cases} w^T\mu + \alpha + u_k \geq 0 \\ u_k \geq 0, \quad x \in X \\ \sum_{i=1}^{n} w_i = 1 \\ 0 \leq w_i \leq 1 \end{cases}$$

where $X = \left\{(x_1, \ldots, x_n) \middle| \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \forall j \in \{1, \ldots, n\}\right\}$. In that system Constraints show that we can manage our capital to invest in several assets. Therefore, the aim of implementation of ACO algorithms consists of finding the Pareto solutions based on the above constraints.

4.2. Simulated Pseudocode:

In this step, we show procedure of performance of the above system by simulating objectives pseudocode:

Choose one initial solution $s_0 \in \Omega$

s**do**

$c = 0;$

$x_j \geq 0;$

A**do**

$\sigma^2(R_x) = \mathrm{w}^{\mathrm{T}}\sigma \mathrm{w};$

$s_1 = \quad \left\{\min_{\mathrm{v}\in \mathrm{R}} \frac{1}{\alpha} E\{\max\{[-R_x, +v], 0\}\} - v\,, -\mathrm{w}^{\mathrm{T}}\mu, \mathrm{w}^{\mathrm{T}}\sigma \mathrm{w}\right\};$

replace s_0 by s_1 ;

$c = c + 1;$

uuntil $c = k;$

quntil $s_1 = s_0;$

$s_0 = solution$

Where variables that used in above pseudocode are according to subsection of 4.1.

4.3. ACO Procedure for MVC

According to section 3 we will present a MVC model of portfolio optimization based on Doerner et al. (2004) Pareto ACO algorithm as the following:

Step1. Initialized by Ant

In the initialization step, let Γ , $x = (0,0, \dots, 0)$ as ants are considered and empty portfolio respectively. For each ant we denote lifespan by Δ and the objective weights via $p = (p_1, p_2, \dots, p_k)$ that they are chosen randomly or one number in $[0,1)$.

Step2. Evaluate the performance

This step refers to evaluation of the algorithm. Also, it includes the local updating for ant via (4).

Step3. Ant distribution

In this step we want to make construction of portfolio x via using the heuristic awareness.

Step4. Stopping the iteration

The process of the ACO algorithm when finishes that we find the best choice for the portfolio x for each objective function k. To sum up, the ACO algorithm for Mean-Variance-CVaR model of portfolio optimization is as the following:

5. Conclusion

The aim of portfolio optimization is finding the minimum of risk and maximum of return in investing.in this paper presented a multi-portfolio model which includes mean, variance, CVaR as new approach based on ACO. In addition, One of the main methods of solving the multi-objective problems is using via algorithms (e.g. Genetic, Bee, Swarm and ACO algorithm). The approach of this study is ACO. This approach is made using three different parameters which discussed in literature, however the lack of ACO model leads to this investigation. In the proposed model, there is one multi-objective system for portfolio optimization that consist of three objective functions. The first objective function is to minimize the conditional value at risk for portfolio optimization. The second objective function denotes minimizing the expected value of return of the portfolio. And the last objective function lets the minimizing the variance of returns. Manage between them help to find the Pareto optimal portfolio optimization.

The proposed algorithm is flexible and extendable in similar cases and model of the new environment and conditions by extending. As the future study, the researchers can try to find the new algorithm for multi-portfolio optimization as the Hybrid method in optimization.

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